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1991 J. Phys.: Condens. Matter 3 215

(http://iopscience.iop.org/0953-8984/3/2/008)

J. Phys.: Condens. Matter 3 (1991) 215–220. Printed in the UK

LETTER TO THE EDITOR

Phase-driven current and quantum interference in the quantum Hall regime of a narrow two-dimensional electron gas

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Received 7 August 1990

Abstract. Novel periodic oscillations of magnetoresistance observed in a recent quantum Hall effect experiment on a narrow two-dimensional sample have been explained on the basis of a Josephson-type effect. The calculated values of the period of oscillation in the regions of plateaux 2 and 4 agree excellently with the measured values.

Some time ago it was suggested by this author that in a narrow two-dimensional electron-gas (2DEG) system subjected to the quantum Hall conditions, a phase-driven alternating current (AC) should flow in the direction transverse to that of the system current [1]. We show here that the prediction of this Josephson-type effect in the quantum Hall regime is confirmed by a recent experimental result of Mottahedeh *et al* [2]. It has already been shown that the quenching of the Hall effect in quasi-1D systems reported by Roukes *et al* [3] is the low-frequency manifestation of the above mentioned Josephson-type effect [4].

Under quantum Hall conditions the system current in a 2DEG consists of two 'edge currents' flowing parallel and anti-parallel to, say, the x-direction. If the 2DEG is sufficiently narrow in the y-direction (the magnetic field $\boldsymbol{B}$ being in the z-direction) then the two edge currents come close to each other and may couple weakly. In the presence of long-range phase order, it was shown that the phase slip between the wavefunctions of the edge currents (represented as $\psi \sim C(r)e^{i\alpha}$, α being the phase) can give rise to an AC in the y-direction [1]. The Hall voltage, V_H (in the y-direction), causes the phase slippage with frequency [1]

$$\dot{\varphi}_{12} = 2\pi V_H/\Phi_0 \tag{1}$$

where φ_{12} is the phase difference between the two edge currents and $\Phi_0 = hc/e$ is the flux quantum. The AC is given by [1]

$$J = J_c \sin \varphi_{12} \tag{2}$$

where the current density J_c is the critical value of J at $\varphi_{12} = \pi/2$. Suppose the narrow

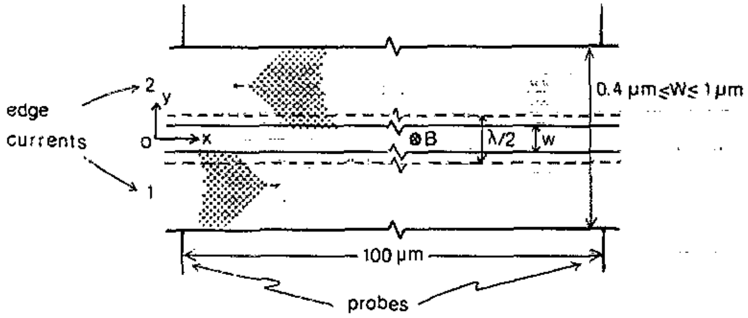

Figure 1. The narrow 2DEG subjected to quantum Hall conditions: the edge currents are shown to be very narrowly separated from each other by the Hall voltage across a region of width w. Coupling of the edge currents gives rise to the AC which flows over a region of width $\lambda/2$ (λ being the wavelength of AC), where $\lambda/2 > w$.

region that separates the edge currents is of width w (figure 1); then the AC flows over a region about $\lambda/2$ wide if $\lambda/2 \geqslant w$ for the given Hall voltage V_H; λ is the wavelength of the AC:

$$\lambda = \Phi_0/V_H. \tag{3}$$

The V_H is developed across the region of width w to stop the flow of electrons from one edge current to the other under the influence of a Lorentz force.

Mottahedeh *et al* [2] carried out the quantum Hall effect (QHE) experiments on narrow 2DEG systems varying the width over a range of about 1 μm down to 0.4 μm. The distance between the voltage probes in the x-direction was about 100 μm. The mobility was about 6400 cm^2 V^{-1} s^{-1}. As the system was narrowed below 1 μm two features began to show up:

(i) in a clear departure from the standard QHE result the magnetoresistance R_{xx} became non-zero in the ranges of B where the Hall resistance R_H ($\equiv R_{xy}$) showed plateaux;

(ii) for systems less than 0.6 μm wide very prominent and *periodic* oscillations developed in the minima of R_{xx}—the period of oscillations was $\Delta B = 0.065$ T for Bs corresponding to a plateau $i = 2$ and was reduced by a factor of two to 0.033 T in the region of $i = 4$. The oscillations were not seen for higher values of i.

Result (i) clearly indicates that the transfer of electrons between the two edge currents gives rise to a non-zero V_{xx} even in the region where V_H ($\equiv V_{xy}$) has plateaux. While under ideal QHE conditions in wide samples the system current I_x is driven *entirely* by V_{xy} (and V_{xx} does not develop), here I_x is driven by both V_{xy} and V_{xx} such that V_{xy}/I_x remains constant. The transfer of electrons between the edge currents can arise due to elastic scattering from impurities as well as due to the phase-driven AC discussed here. Result (ii), it is argued here, arises out of quantum interference that occurs when the edge currents are mixed by the AC.

A number of studies have recently been devoted to quantum interference effects in narrow 2DEG systems [5]. In almost all of these the communication between the edge currents is taken as having been established by scattering from impurities or by resonant tunnelling through a bound state on an impurity. Such considerations cannot explain the periodic oscillations because more than one such event involving impurities of different sizes will erode a well defined periodicity.

The current-carrying electrons move in opposite directions along the edge. Some of these go back and forth between the edge currents under the influence of the phase slippage and end up moving in closed loops. Assuming that when the electrons enter the sample they all have the same phase at all times, that the elastic scattering events are too few and too weak to make an appreciable change in the trajectories of the electrons, and that the inelastic scattering events are negligible in the mK region of temperature in which the experiments are done, we can take the areas enclosed in each loop to be the same. As the electrons move along the closed trajectories their phase changes due to the magnetic field $\boldsymbol{B}$, and the Hall voltage V_H. We will show that the phase difference φ_{12} between the edge currents 1 and 2 changes by 2π as the electrons go around in a loop once. This enables us to calculate ΔB, the periodicity of the observed oscillations in R_{xx}. The calculated values of ΔB corresponding to plateaux $i = 2$ and 4 agree exactly with the observed values.

We will analyse the variation in the quantum-mechanical phase of the current-carrying electrons after they enter into the Hall device. First we will work out the phase change introduced by $\boldsymbol{B}$ and then we will include the effect of V_H.

Taking $\boldsymbol{B}$, which is in the z-direction, to be uniform across and around the sample, we choose a convenient gauge in which the vector potential has no y-component and is given by $\boldsymbol{A} = [A_x(y), 0, 0]$:

$$A_x(y) = By \qquad -\infty \leqslant y \leqslant \infty \tag{4}$$

where $y = 0$ is along the middle of the device. Now recall the gauge-invariance expression relating the phase gradient to the canonical momentum,

$$\boldsymbol{\nabla}\alpha = 2m\boldsymbol{v}_s/\hbar + (e/\hbar c)\boldsymbol{A}. \tag{5}$$

The phase difference between points a and b on the same edge current is

$$\alpha(b) - \alpha(a) = \frac{2m}{\hbar}\int_a^b \boldsymbol{v}_s \cdot \mathrm{d}\boldsymbol{w} + \frac{e}{\hbar c}\int_a^b \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{w}. \tag{6}$$

Note that since $\boldsymbol{A}$ and $\boldsymbol{v}_s$ have only x-components, it is clear that if the points a and b have the same x-coordinate then they will have the same phase (for $x_a = x_b$, $\mathrm{d}\boldsymbol{w}$ will always be perpendicular to the x-direction, so the above integrals vanish). Consequently on each side of the device we have from (6)

$$\begin{aligned} \alpha_1(x, -\infty) &= \alpha_1(x, -\lambda/4) = \alpha_1(x, -w/2) \\ \alpha_2(x, \infty) &= \alpha_2(x, \lambda/4) = \alpha_2(x, w/2). \end{aligned} \tag{7}$$

The subscripts 1 and 2 represent the two sides of the device. The z-variable is ignored since the phase is independent of z in a 2DEG lying in the xy plane.

The above analysis helps in calculating the phase difference across the device (between the two sides) which is given by

$$\varphi_{12}(x) = \alpha_2(x, y) - \alpha_1(x, -y) - \frac{e}{\hbar c}\int_1^2 \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{w}. \tag{8}$$

Since the AC is flowing over a width of $\lambda/2$ about the $y = 0$ line, we need to calculate $\varphi_{12}(x)$ between points lying on two parallel lines $\lambda/4$ away from the $y = 0$ axis on either side of it; moreover in the gauge we have chosen, A is perpendicular to $\mathrm{d}w$, so the integral in (8) vanishes and we need only calculate

$$\varphi_{12} = \alpha_2(x, \lambda/4) - \alpha_1(x, -\lambda/4) = (\alpha_2(x, \lambda/4) - \alpha_2(0, \lambda/4)) - (\alpha_1(x, -\lambda/4) - \alpha_1(0, -\lambda/4)) + \varphi_{12}^0 \tag{9}$$

where $\varphi_{12}^0 = \alpha_2(0, \lambda/4) - \alpha_1(0, -\lambda/4)$. Using (6) we obtain

$$\alpha_2(x, \lambda/4) - \alpha_2(0, \lambda/4) = \frac{2m}{\hbar}\int_0^x (-v_x)\,\mathrm{d}x + \frac{e}{\hbar c}\int_0^x A_x(\lambda/4)\,\mathrm{d}x \tag{10a}$$

$$\alpha_1(x, -\lambda/4) - \alpha_1(0, -\lambda/4) = \frac{2m}{\hbar}\int_0^x v_x\,\mathrm{d}x + \frac{e}{\hbar c}\int_0^x A_x(-\lambda/4)\,\mathrm{d}x. \tag{10b}$$

The velocities, v_x, of charge carriers are equal but opposite at two points located symmetrically about $y = 0$ for a fixed x. Taking $A_x(\pm\lambda/4)$ from (4) we get

$$\varphi_{12} = \varphi_{12}^0 - (4m/\hbar)v_x x + 2\pi B(\lambda/2)x/\Phi_0. \tag{11}$$

This gives the x-dependence of the phase difference between two lines that are $\lambda/2$ apart and lie in the two edge currents, arising due to the currents flowing with velocity v_x (second term on RHS) and due to the magnetic field $\boldsymbol{B}$ (third term on RHS).

However, an additional change, with x, in the phase difference, φ_{12}, occurs due to the presence of V_{H} across the width w as an electron enters into the edge current 1 and moves a distance x. If the distance x is travelled in time t, then the *total* change in the phase difference that occurs between the above mentioned two lines as an electron moves a distance x is

$$\varphi_{12}(x, t) - \varphi_{12}^0 = -(4m/\hbar)v_x x + 2\pi B(\lambda/2)x/\Phi_0 + eV_{\mathrm{H}}t/\hbar \tag{12a}$$

$$\varphi_{12}(x) - \varphi_{12}^0 = -(4m/\hbar)v_x x + (2\pi/\Phi_0)(B\lambda/2 + V_{\mathrm{H}}/v_x)x. \tag{12b}$$

The first term on the right-hand side of (12b) makes a negligible contribution compared with that made by the second term—e.g. for a distance x, $\sim 0.5\ \mu\mathrm{m}$ (calculated later), over which the second term contributes π, the contribution of the first term is smaller by more than two orders of magnitude; we will ignore this term. To estimate $\varphi_{12}(x)$ we will take the classical value for v_x which is V_{H}/wB. Taking $w \approx \lambda/2$, we get

$$\varphi_{12}(x) = \varphi_{12}^0 + (4\pi/\Phi_0)B\lambda x/2. \tag{13}$$

Now we can calculate the φ_{12} that develops as certain electrons complete a loop after the AC is set up between the edge currents.

Suppose $\varphi_{12}^0 = \pi/2$ when $x = 0$, i.e., when an electron enters at the left-hand end of the device into the edge current 1. Then $\varphi_{12} = \pi/2$, and according to (2) a maximum current of magnitude J_c is flowing from side 1 to side 2 [1]. In this situation, since the electrons are flowing from side 2 to side 1, the electron that entered at $x = 0$ will continue to move in the x-direction. As it is moving, φ_{12} is continuously changing in accordance with equation (13). When φ_{12} becomes $3\pi/2$, J would be $-J_c$, i.e., a maximum current of magnitude J_c would flow from side 2 to side 1. At this stage the electron under consideration would probably move from edge current 1 to 2†. Up until now φ_{12} has changed by π. On joining the edge current 2 the electron moves in the negative x-direction and if v_s is the same as it was in the edge current 1, it will rise to $x = 0$ when J will become $+J_c$ and will return to where it originally started, thus completing the loop. φ_{12} has by now changed by 2π; alternatively we can say that the electron has gone back to the origin undergoing a phase change of 2π relative to the freshly injected electrons at this instant whose phase is $\pi/2$.

To calculate ΔB, the periodicity of the oscillations under consideration, suppose that there are n closed loops placed side by side along the length L of the system. In this situation the net AC flowing between the edge currents will be zero because $J = J_c$ at $x = 0$, and at $x = L$, $J = +J_c$ or $-J_c$ (depending on whether n is even or odd). The net AC will be zero again as n goes to $n + 1$ for B, enclosed in the rectangle $\lambda/2 \times L$, increasing; in this case J_c, at $x = L$, will change its sign compared with whatever it was for n. The change in B that takes n to $n + 1$ is, in fact, the ΔB we want to calculate—this is the separation between two consecutive minima of R_{xx}. Whenever the net AC is zero, the system current in the longitudinal direction will be at its maximum and therefore the R_{xx} will be at its minimum. When the AC is non-zero the magnitude of the system current is lowered, which makes R_{xx} larger compared with its value when AC was zero. Thus as the AC oscillates between zero and $\pm J_c$, the R_{xx} oscillates between its minimum and maximum values. The separation, ΔB, between two consecutive minima of R_{xx} therefore corresponds to two consecutive zeros of AC corresponding to the n and $n + 1$ loops.

At a given B, that gives rise to n loops fitted in the length L, the change in phase over the length L will be $n\pi$ as seen above. Then according to equation (13)

$$(4\pi/\Phi_0)B(\lambda/2)L = n\pi \tag{14}$$

and when B increases by ΔB, making n increase to $n + 1$,

$$(4\pi/\Phi_0)(B + \Delta B)(\lambda/2)L = (n + 1)\pi. \tag{15}$$

Therefore,

$$\Delta B = \Phi_0/2\lambda L = V_H/2L \tag{16}$$

using λ from equation (3). For plateau $i = 2$, $V_H = 12.9065 \times 10^{-6}$ V, so, for $L = 100\ \mu$m,

$$\Delta B = 0.0645\ \mathrm{T}$$

which is in excellent agreement with the observed value of 0.065 T. Further, the V_H corresponding to $i = 4$ is exactly half of its value for $i = 2$, so the ΔB in the region of $i = 4$ should be exactly one half of its value for $i = 2$, i.e., it should be about 0.0323 T, again in excellent agreement with the experiment. Thus, the V_H-dependence of ΔB as found in equation (16) is in conformity with the experiment.

The excellent agreement of the calculated and the measured values of ΔB renders undoubtable support to the proposed existence of phase-driven AC in the narrow quantum Hall samples. But there is a need to review it in the light of the assumption (ii) (indicated at the beginning) which is central to the observation of the periodic oscillations. There are two things to be noted in this connection:

† The transverse current from side 2 to side 1 will actually start flowing as soon as φ_{12} exceeds the value π, and with this the probability for an electron to move from edge current 1 to 2 will become non-zero. But this probability will be maximum when the transverse current reaches its maximum. We have taken these points of maximum probability of the transition between the two edge currents as the turning points for the formation of the loops.

(i) the dimensions of a loop: the x-dimension, which is the same as the distance over which φ_{12} changes by π, is given by

$$4\pi B(\lambda/2)x/\Phi_0 = \pi \qquad x = \Phi_0/2B\lambda = V_H/2B = 0.5\,\mu\mathrm{m}$$

where V_H corresponds to plateau $i = 2$, and the y-dimension, namely $\lambda/2$, is $\sim 0.00015\,\mu$m; and

(ii) for the mobility of the given sample, the elastic mean free path is about $0.05\,\mu$m [6].

Assuming that the impurities (the elastic scattering centres) are distributed uniformly, the narrow strip of dimensions $100\,\mu\mathrm{m} \times 0.00015\,\mu\mathrm{m}$ over which the AC is flowing may happen to lie with respect to the array of impurities in such a way that the number of impurities encountered per loop may be any number between the minimum of zero and the maximum of ten. It is hard to judge if an average of ten collisions or so per loop is too little or is large enough to make a substantial change in the trajectory of a loop, because we do not know anything about the strength of the scatterers. We can, though, easily say that in the sample whose results we have discussed here, the $100\,\mu\mathrm{m} \times 0.00015\,\mu\mathrm{m}$ strip lies such that either no impurity falls on it, or very few impurities of negligibly weak strength are encountered.

Thanks are due to M Pepper and R Mottahedeh for discussing their results with me before publication. Hospitality at the Cavendish Laboratory (Cambridge, UK) where this work was begun is gratefully acknowledged.

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